Parabola equations

In this notebook you will find many examples of graphs of parabolas. The idea is to experiment and get some idea of the kinds of shapes you can expect, and how these shapes are related to the basic equation. The kinds of parabolas you will see in this lesson ALL have equations of the form $y = a x^2 + b x + c$

This is just an INTRODUCTION. Form some hypotheses, make a few notes, think of some questions. Experiment with the input lines and change them to see the result. The idea is to get some idea of the effect of the values of a, b and c on the graphs of the equation $y = a x^2 + b x + c$

There are a few programs that are needed for this notebook. You DON'T need to know how they work or how they were written. You don't even have to look at them. Simply select the Programs cell below and evaluate it. Then go on!

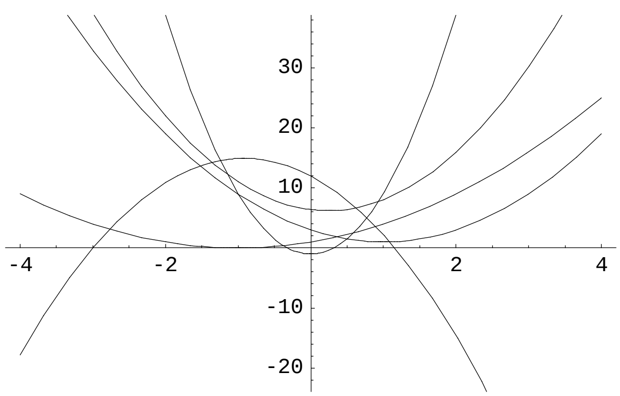
■ Programs

■ Which parabola is it that is opening up instead of down?

$$\mathbf{q1 = x^2 + 2\,x + 1;}$$
$$\mathbf{q2 = 2\,x^2 - 4\,x + 3;}$$
$$\mathbf{q3 = p\,x^2 - \frac{3\,x}{2} + 6.4;}$$
$$\mathbf{q4 = Expand\left[-\left(\sqrt{12.}\;x - 4\right)(x + 3)\right];}$$
$$\mathbf{q5 = 10\,x^2 - 1;}$$

q1 - q5 are 5 parabolas. They are all pretty much the same, except one equation has a small difference that you can see in the graphs below.

```
Plot[{q1, q2, q3, q4, q5}, {x, -4, 4}];
```



If you can determine the difference, modify some of the other equations and see if you can make them open down as well.

■ Teacher note

The difference is that q4 has a negative value of a

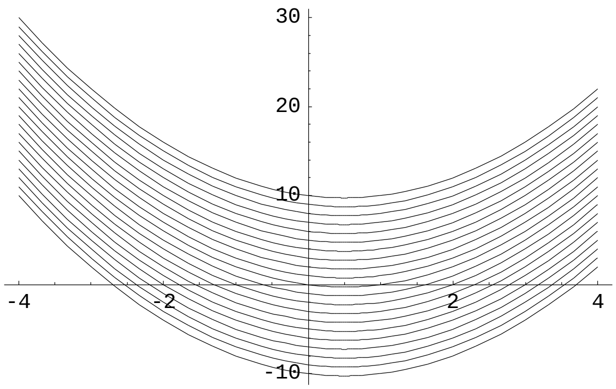
■ Explain the algebra and the geometry of the parabolas shown here.

```
TableForm[ Partition[
    Table[x^2 - x + c, {c, -10, 10}],
3] ]
```

$x^2 - x - 10$	$x^2 - x - 9$	$x^2 - x - 8$
$x^2 - x - 7$	$x^2 - x - 6$	$x^2 - x - 5$
$x^2 - x - 4$	$x^2 - x - 3$	$x^2 - x - 2$
$x^2 - x - 1$	$x^2 - x$	$x^2 - x + 1$
$x^2 - x + 2$	$x^2 - x + 3$	$x^2 - x + 4$
$x^2 - x + 5$	$x^2 - x + 6$	$x^2 - x + 7$
$x^2 - x + 8$	$x^2 - x + 9$	$x^2 - x + 10$

Above you can see a table with many expressions. Examine them to see what is the same and what is different. Look for patterns. Next, examine the graphs that these expressions would produce and try to determine what determines what!?

```
Plot[Evaluate[%], {x, -4, 4}];
```



■ Teacher note

The value of c determines the "up" and down location of the parabola

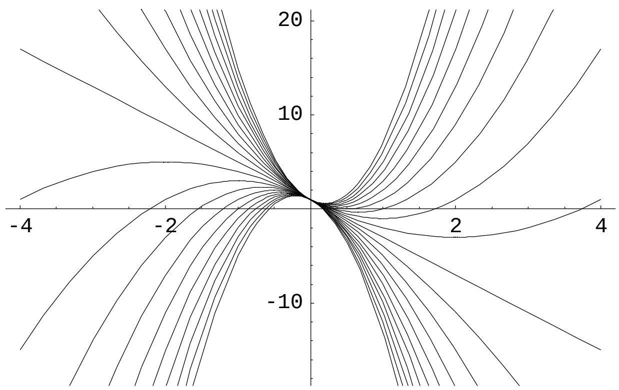
■ What makes a parabola point up or down?

```
TableForm[ Partition[
    Table[a x^2 - 4 x + 1, {a, -10, 10}],
3] ]
```

$1 - 4x - 10x^2$	$1 - 4x - 9x^2$	$1 - 4x - 8x^2$
$1 - 4x - 7x^2$	$1 - 4x - 6x^2$	$1 - 4x - 5x^2$
$1 - 4x - 4x^2$	$1 - 4x - 3x^2$	$1 - 4x - 2x^2$
$1 - 4x - x^2$	$1 - 4x$	$1 - 4x + x^2$
$1 - 4x + 2x^2$	$1 - 4x + 3x^2$	$1 - 4x + 4x^2$
$1 - 4x + 5x^2$	$1 - 4x + 6x^2$	$1 - 4x + 7x^2$
$1 - 4x + 8x^2$	$1 - 4x + 9x^2$	$1 - 4x + 10x^2$

Above you can see another table with many expressions. Examine them to see what is the same and what is different. Look for patterns. Next, examine the graphs that these expressions would produce and try to determine what determines what!?

```
Plot[Evaluate[%], {x, -4, 4}];
```



■ Teacher note

The value of a determine whether a parabola opens up or down, and also how narrow or wide it is. Note as well that one of the equations is NOT a parabola, (1 - 4x) , it creates the line

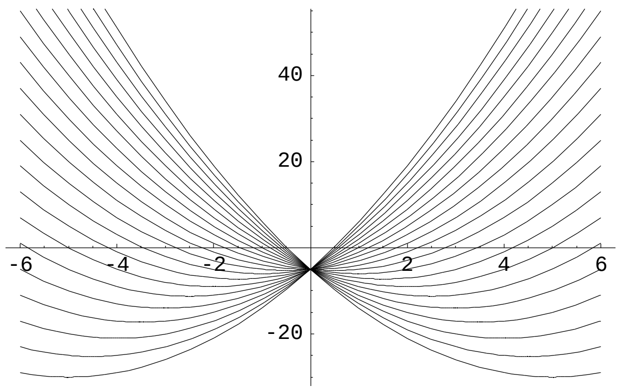
■ Why do all these parabolas go through the same point?

```
TableForm[ Partition[
    Table[x^2 +  b x - 5, {b, -10, 10}],
    3] ]
```

$x^2 - 10x - 5$	$x^2 - 9x - 5$	$x^2 - 8x - 5$
$x^2 - 7x - 5$	$x^2 - 6x - 5$	$x^2 - 5x - 5$
$x^2 - 4x - 5$	$x^2 - 3x - 5$	$x^2 - 2x - 5$
$x^2 - x - 5$	$x^2 - 5$	$x^2 + x - 5$
$x^2 + 2x - 5$	$x^2 + 3x - 5$	$x^2 + 4x - 5$
$x^2 + 5x - 5$	$x^2 + 6x - 5$	$x^2 + 7x - 5$
$x^2 + 8x - 5$	$x^2 + 9x - 5$	$x^2 + 10x - 5$

Above you can see yet another table with many expressions. Examine them to see what is the same and what is different. Look for patterns. Next, examine the graphs that these expressions would produce and try to determine what determines what!?

```
Plot[Evaluate[%], {x, -6, 6}];
```



■ Teacher note

The value of c is the same in each parabola, and this value will be the *y*-intercept of the parabola. In this case, each parabola has a *y*-intercept of -5. The value of "b" has an effect on where the vertex of the parabola is located.

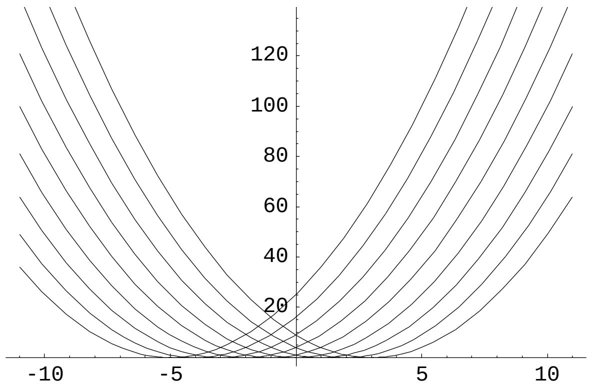
■ Moving the vertex

```
TableForm[ Partition[
    Table[(x - k)^2, {k, -5, 5}],
    3] ]
```

$(x+5)^2$	$(x+4)^2$	$(x+3)^2$
$(x+2)^2$	$(x+1)^2$	x^2
$(x-1)^2$	$(x-2)^2$	$(x-3)^2$

Examine the table of expressions and look for patterns. How do these expressions relate to the graphs shown below?

```
Plot[Evaluate[%], {x, -11, 11}];
```



■ Teacher note

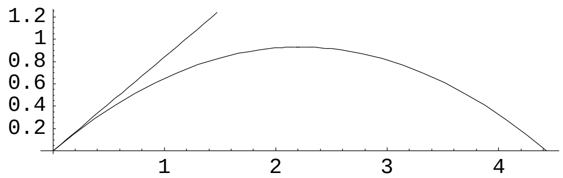
The vertex of the parabola $y = (x - k)^2$ is at $(0, k)$

■ Projectiles

These examples are related to physics and projectiles. A projectile will follow the path of a parabola if there is no air resistance. Experiment with various starting values to see the effect on the parabola produced.

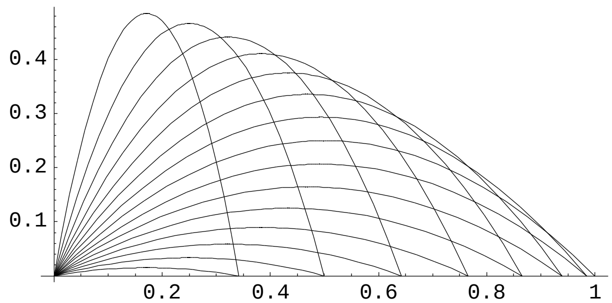
- **The path of a projectile depends on the gravity, initial velocity, and angle.**

```
Projectile[2, 3, 40];
```



- **What angle gives the longest horizontal distance?**

```
Projectiles[1, 1, Range[10, 80, 5]];
```



- **This varies the initial velocity.**

```
Projectiles[1, Range[10, 80, 5], 38];
```